Models In Epidemiology And Biostatistics
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The Odds Ratio Is Larger Than The Risk Ratio

The odds ratio [OR] is always further away from the null [1] than the risk ratio [RR].

Lets check this. First, the notation :

Lets write p=Pr(D) so that $p_1 = Pr(\mathrm{D} \mid \mathrm{E})$ $p_0 = Pr(\mathrm{D} \mid \bar{\mathrm{E}})$ then :

the odds ratio is $\mathrm{OR} = \dfrac{\dfrac{p_1}{1-p_1}}{\dfrac{p_0}{1-p_0}}$ the risk ratio is $\mathrm{RR} = \dfrac{p_1}{p_0}$ and the 'health' ratio is $\mathrm{HR} = \dfrac{1-p_1}{1-p_0}$

$$\mathrm{OR} = \frac{\dfrac{p_1}{1-p_1}}{\dfrac{p_0}{1-p_0}} = \frac{\dfrac{p_1}{p_0}}{\dfrac{1-p_1}{1-p_0}} = \frac{\mathrm{RR}}{\mathrm{HR}}$$

Check out both cases :

Case 1: If : $\mathrm{RR} = \dfrac{p_1}{p_0} > 1$ then $p_1 > p_0$ or $1-p_1 < 1-p_0$ so $\mathrm{HR} = \dfrac{1-p_1}{1-p_0} < 1$

So $\mathrm{OR} = \dfrac{\mathrm{RR}}{\mathrm{HR}} > \mathrm{RR}$

If RR >1 then OR > RR.

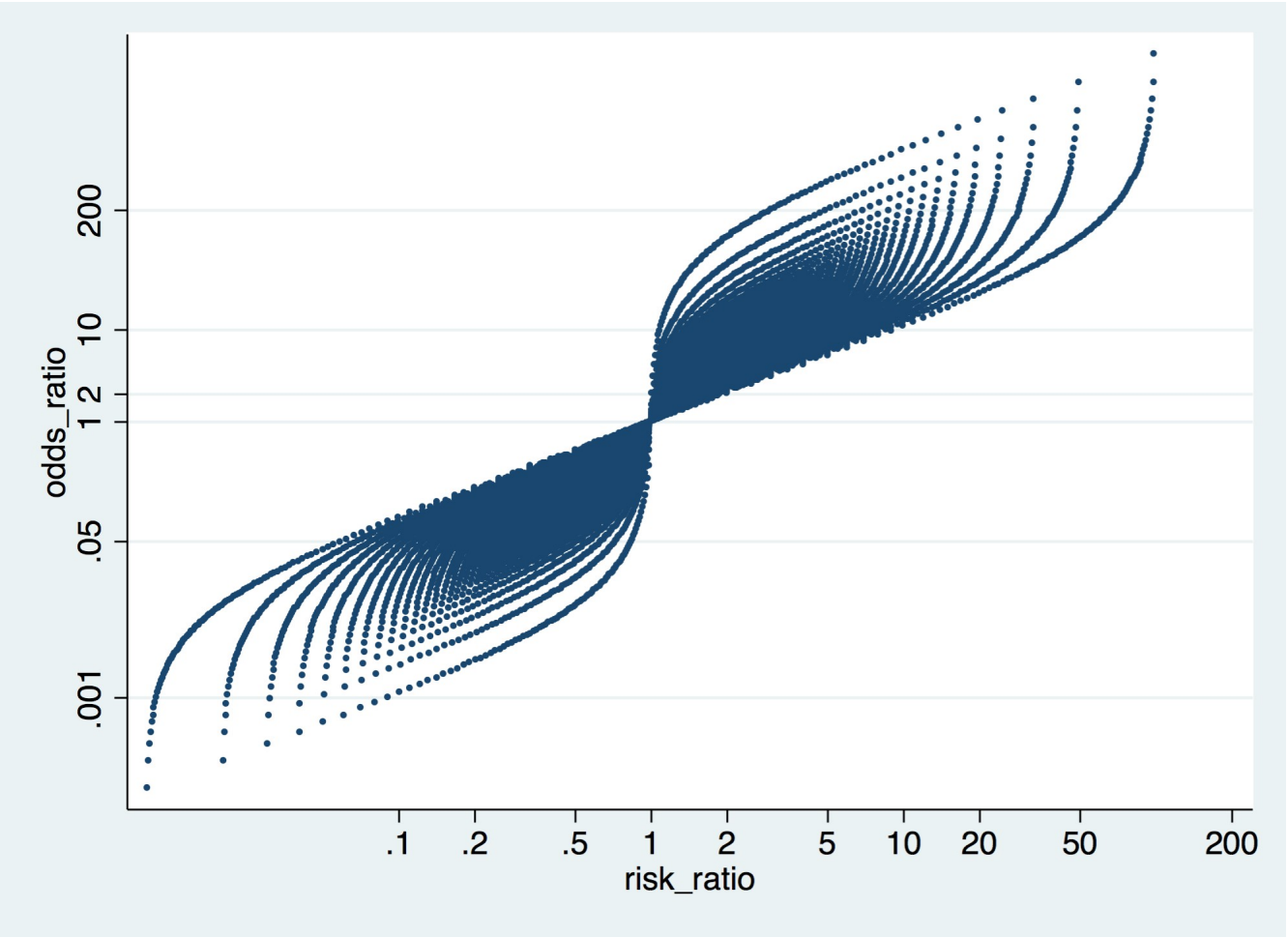
Case 2: If : $\mathrm{RR} = \dfrac{p_1}{p_0} < 1$ then $p_1 < p_0$ or $1-p_1 > 1-p_0$ so $\mathrm{HR} = \dfrac{1-p_1}{1-p_0} > 1$

So $\mathrm{OR} = \dfrac{\mathrm{RR}}{\mathrm{HR}} < \mathrm{RR}$

If RR <1 then OR < RR.

Notice that $\dfrac{\mathrm{RR}}{\mathrm{OR}} = \mathrm{HR}$ or $\mathrm{RR} = \mathrm{HR} * \mathrm{OR}$. So the RR is the same as the OR only when the HR = 1.

One can graph the OR versus the RR. [using or_rr.do]
The OR can be quite close to the RR and they can be quite a distance from each other.



For the estimates of the RR and the estimates of the OR, it is a more mixed bag.

We can next see that the crude estimates satisfy the inequality too.
So do the stratum specific estimates.

Is this true for the 'adjusted' estimates? Not all the time?
For the 'classical methods', MH weights for the adjusted OR estimate are different from the MH weights for the adjusted RR estimate.
If the stratum specific RRs are all greater than one, then the inequality holds. Same if all RRs are less than one.
For models based on logit link and models based on the log link, I think this is an unsolved question.